A General Formula for Black Hole Gravitational Wave Kicks

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ABSTRACT

Although the gravitational wave kick velocity in the orbital plane of coalescing black holes has been understood for some time, apparently conflicting formulae have been proposed for the dominant out-of-plane kick, each a good fit to different data sets. This is important to resolve because it is only the out-of-plane kicks that can reach more than 500 km $s^{-1}$and can thus eject merged remnants from galaxies. Using a different ansatz for the out-of-plane kick, we show that we can fit almost all existing data to better than 5%. This is good enough for any astrophysical calculation, and shows that the previous apparent conflict was only because the two data sets explored different aspects of the kick parameter space.

Subject headings: black hole physics — galaxies: nuclei — gravitational waves

1. Introduction

When two black holes spiral together and merge, the gravitational radiation they emit is usually asymmetric and thus the remnant black hole acquires a linear velocity relative to the original center of mass. The speed can reach over 3,000 km s^{-1}(Dain et al. 2008), which would eject the remnant from any galaxy in the universe (see Figure 2 of Merritt et al.

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(2004)). As discussed in Merritt et al. (2004), the kick magnitude and distribution are important for discussions of hierarchical merging, supermassive black hole formation, galactic nuclear dynamics, and the degree to which black holes influence galaxy formation. The community has converged on the formula for the kick speed in the original orbital plane (Baker et al. 2007; Campanelli et al. 2007b; Gonzalez et al. 2007), but apparently conflicting dependences for the out-of-plane kick (which dominates the total kick for most configurations) have been proposed. Lousto & Zlochower (2009) suggested that for a binary with component masses m_1 and $m_2 \geq m_1$, the kick scales as η^2, where $\eta \equiv m_1 m_2/(m_1 + m_2)^2$ is the *symmetric mass ratio*; however the data in Baker et al. (2008) were fit much better with an η^3 dependence. The conflict is only apparent, however, because there were no runs in common between the two data sets. This suggests that an analysis might be performed with a new ansatz that can fit all of the existing data.

Here we perform such a fit, and demonstrate that there is a single formula for the out-of-plane kick that fits almost all existing data to better than 5% accuracy. The new ansatz is similar to one recently suggested by Lousto et al. (2009), for which, however, a fit was not attempted. Its form is based straightforwardly on the post-Newtonian (PN) approximation and includes both the aforementioned η^2 and η^3 terms, as well as a slightly more complicated spin-angle dependence. In § 2 we describe our new runs and we describe the new ansatz in § 3. In § 4 we list all 95 runs we have fit, from different numerical relativity groups, and our fitting procedure and best-fit parameters. In § 5 we discuss the implications of our results, and indicate the fraction of kicks above 500 km s^{-1}and 1,000 km s^{-1}for representative spin and mass ratio distributions, comparing it with previous results. The goodness of our fit to the entire usable data set of out-of-plane kicks suggests that the full three-dimensional kick is now modelled well enough that it will not limit the accuracy of any astrophysical calculation.

2. Numerical simulations

We have performed new simulations representing 22 distinct physical cases. Defining $q \equiv m_1/m_2$ and $\alpha_i \equiv S_i/m_i^2$ where S_i is the spin angular momentum of the ith black hole, we used mass ratios of $q = 0.674$ or $q = 0.515$, and spins initially within the orbital plane of $\alpha_1 = \alpha_1^{\perp} = 0.367$ and $\alpha_2 = \alpha_1^{\perp} = 0.177$ or $\alpha_2 = \alpha_1^{\perp} = 0.236$, where the $\perp$ superscript indicates orthogonality to the orbital axis. For each mass ratio we used one of eleven different spin orientations (given in Table 1), in order to probe the spin-angle dependence of the final recoil. Initial parameters were informed by a quasicircular PN approximation and initial data constructed using the spectral solver `TwoPunctures` (Ansorg et al. 2004). Evolutions were

performed with the Einstein-solver `Hahndol` (Imbiriba et al. 2004; van Meter et al. 2006; Baker et al. 2007), with all finite differencing and interpolation at least fifth-order-accurate in computational grid spacing. Note for the purpose of characterizing the simulations, it is convenient to use units in which $G = c = 1$ and specify distance and time in terms of $M \equiv m_1 + m_2$. The radiation field represented by the Weyl scalar Ψ_4 was interpolated to an extraction sphere at coordinate radius $r = 50M$ and integrated to obtain the radiated momentum using the standard formula (Schnittman et al. 2008):

$$P_i = \int_{-\infty}^{t} dt \frac{r^2}{16\pi} \int d\Omega \frac{x_i}{r} \left| \int_{-\infty}^{t} dt \Psi_4 \right|^2 . \tag{1}$$

For each of the 22 cases we ran two resolutions, with fine-grid spacings of $h_f = 3M/160$ and $h_f = M/64$. To give some indication of our numerical error, the total recoil from the two resolutions for each of the $q = 0.674$ cases agreed to within $\lesssim 6\%$, and for each of the $q = 0.515$ cases to within 1% (the latter using a more optimal grid structure).

In these simulations, variation in the magnitude of the final recoil within the x-y plane suggested non-negligible precession of the orbital plane. Indeed the coordinate trajectories of the black holes showed precession of up to $\sim 10°$. To calculate the component of the recoil velocity parallel to the "final" orbital axis, $V_{||}$, we tried two different methods. In one method we simply took the dot product of the final, numerically computed recoil velocity $\mathbf{V}$ with the normalized orbital angular momentum $\mathbf{L}|L|^{-1}$ as calculated from the coordinate trajectories of the black holes, just when the common apparent horizon was found: $V_{||} \equiv \mathbf{V} \cdot \mathbf{L}|L|^{-1}$. In our second method we assumed that each black hole spin, which is initially orthogonal to the orbital angular momentum, remained approximately orthogonal throughout the simulation, i.e.

$$\mathbf{L} \cdot \mathbf{S}_1 \approx \mathbf{L} \cdot \mathbf{S}_2 \approx 0. \tag{2}$$

This assumption is consistent with PN calculations, to linear order in spin (Racine 2008; Kidder 1995), and is also supported numerically by the fact that the merger times in our simulations were independent of the initial spin, to within $\Delta t < 1M$. In this case the in-plane recoil should depend only on the mass ratio (and not the spin), and we assume it is given by the formula found by Gonzalez et al. (2007), with coefficient values given by a previous fit (Baker et al. 2008). This implies

$$V_{||} \equiv \sqrt{V^2 - (V_{\perp}^{pred})^2} \frac{V_z}{|V_z|}, \tag{3}$$

where we have further assumed that the sign of $V_{||}$ should be the same as that of V_z, given the modest amount of precession.

These two definitions for $V_{||}$ were found to differ by a relative error of $\lesssim 5\%$ for all points except one for which $V_{||} \ll V_{\perp}$. Relative to the maximum $V_{||}$ per mass ratio, they were found

to differ only by $< 2\%$. Note this lends further support to our assumption that the spins are orthogonal to the orbital angular momentum to good approximation throughout these simulations (Eq. (2)). Even V_z was found to differ from the above definitions for $V_{||}$ by only $\lesssim 2\%$, relative to the maximum. We will use Eq. (3) to give our canonical $V_{||}$, for the purpose of analytic fits to the data.

An additional assumption we will make about our data, important for fitting purposes, is that the amount of precession undergone by the spins in the orbital plane, i.e. the difference in spin-angles, between the initial data and the merger, is independent of the initial spin orientation. This assumption, valid to linear order in spin according to the PN approximation (Eq. (2.4) of (Kidder 1995)), was previously found to be the case through explicit computation of the spins in the simulations presented in Baker et al. (2008). We have not explicitly computed the spins in the new simulations presented here but the independence of the merger time with respect to initial spin orientation is consistent with the assumption of similar independence of spin precession.

For the purpose of constructing an accurate phenomenological model, we added to this data set previously published data representing out-of-plane kicks. Our criteria for selecting relevant data are that the component of the final recoil parallel to the orbital plane is given and at least three different in-plane spin orientations are used. We arrived at a total of 95 data points: the 22 new ones presented here plus 73 drawn from Baker et al. (2008); Campanelli et al. (2007b); Dain et al. (2008); Lousto & Zlochower (2009). Note that for the cases in Lousto & Zlochower (2009) we use the second version of their values listed in their Table III, resulting from what they consider to be their best calculation of the final orbital plane.

3. Ansatz

An ansatz for the total recoil that was found to be very consistent with numerical results has the form (Baker et al. 2007; Campanelli et al. 2007b; Gonzalez et al. 2007)

$$\begin{aligned} \vec{V} &= V_{\perp m}\,\mathbf{e}_1 + V_{\perp s}(\cos\xi\,\mathbf{e}_1 + \sin\xi\,\mathbf{e}_2) + V_{\|}\,\mathbf{e}_3, && (4)\\ V_{\perp m} &= A\eta^2\sqrt{1-4\eta}(1+B\eta), && (5)\\ V_{\perp s} &= H\frac{\eta^2}{(1+q)}\left(\alpha_2^{\|} - q\alpha_1^{\|}\right), && (6) \end{aligned}$$

where $\mathbf{e}_1$ and $\mathbf{e}_3$ are unit vectors in the directions of separation and the orbital axis just before merger, respectively, $\mathbf{e}_2 \equiv \mathbf{e}_1 \times \mathbf{e}_3$, $\alpha_1^{\|}$ and $\alpha_2^{\|}$ represent the components of spins parallel to the orbital axis, ξ, A, B, and H are constant fitting parameters, and $V_{\|}$ is to be

discussed.

Every component of Eq. (4) has been modeled to some degree after PN expressions. The use of such PN-based formulae has proven very successful. For example, Eq. (5) for the in-plane, mass-ratio-determined recoil can be obtained from the corresponding PN expression (Eq. (23) of Blanchet et al. (2005)) simply by taking the leading-order terms and replacing instances of the PN expansion parameter (in this case frequency) with constant fitting parameters. Using this, Gonzalez et al. (2007) obtained very good agreement with a large set of numerical results. Le Tiec et al. (2010), who calculated the recoil using a combination of the PN method with a perturbative approximation of the ringdown, also found that Eq. (5) gave a good phenomenological fit to their analytic results.

Why such a prescription for generating an ansatz from the PN approximation should apply so well to merger dynamics is not perfectly understood. The effective replacement of powers of the frequency with constants may be defensable because the majority of the recoil is generated within a narrow time-window near merger, perhaps within a narrow range of frequencies. This is particularly evident for the out-of-plane recoil speed, which rapidly and monotonically increases up to a constant value around merger. However, values of the constant coefficients that appear in the PN expansion cannot be expected to remain unchanged because, as merger is approached, high-order PN terms with the same functional dependence on mass and spin as leading order terms can become comparable in magnitude. So, it is just for this functional dependence that we look to the PN approximation for guidance.

Following the prescription implied above, from the PN expression for out-of-plane recoil given by Racine et al. (2009), Eqs. (4.40-4.42) (neglecting spin-spin interaction), the following can be straightforwardly obtained:

$$\begin{aligned} V_{||} &= \frac{K_2\eta^2 + K_3\eta^3}{q+1}\left[q\alpha_1^{\perp}\cos(\phi_1 - \Phi_1) - \alpha_2^{\perp}\cos(\phi_2 - \Phi_2)\right] \\ &+ \frac{K_S(q-1)\eta^2}{(q+1)^3}\left[q^2\alpha_1^{\perp}\cos(\phi_1 - \Phi_1) + \alpha_2^{\perp}\cos(\phi_2 - \Phi_2)\right], \end{aligned} \tag{7}$$

where K_2, K_3, and K_S are constants, $\alpha_i^{\perp}$ represents the magnitude of the projection of the ith black hole's spin (divided by the square of the black hole's mass) into the orbital plane, ϕ_i represents the angle of the same projection, as measured at some point before merger, with respect to a reference angle representing the direction of separation of the black holes, and Φ_i represents the amount by which this angle precesses before merger, which depends on the mass ratio and the initial separation. We have ignored terms quadratic in the spin because we assume they are subleading. Note that this ansatz is equivalent to one suggested by Lousto et al. (2009), provided the angular parameters are suitably interpreted. In terms

of the notation of Racine et al. (2009),

$$(q+1)^{-1}\left[q\alpha_1^{\perp}\cos(\phi_1-\Phi_1)-\alpha_2^{\perp}\cos(\phi_2-\Phi_2)\right] = -M^{-1}\mathbf{\Delta}\cdot\mathbf{n} = -M^{-1}\Delta^{\perp}\cos(\Theta), \quad (8)$$

$$(q+1)^{-2}\left[q^2\alpha_1^{\perp}\cos(\phi_1-\Phi_1)+\alpha_2^{\perp}\cos(\phi_2-\Phi_2)\right] = M^{-2}\mathbf{S}\cdot\mathbf{n} = M^{-2}S^{\perp}\cos(\Psi), \quad (9)$$

where $\mathbf{n}$ is a unit separation vector, $\Delta \equiv \mathbf{S}_2/m_2 - \mathbf{S}_1/m_1$, $\mathbf{S} \equiv \mathbf{S}_1 + \mathbf{S}_2$, $\Delta^{\perp} \equiv M(q+1)^{-1}|\alpha_2^{\perp} - q\alpha_1^{\perp}|$, $S^{\perp} \equiv M^2(q+1)^{-2}|\alpha_2^{\perp} + q^2\alpha_1^{\perp}|$, Θ is the angle between Δ and $\mathbf{n}$, and Ψ is the angle between $\mathbf{S}$ and $\mathbf{n}$, all measured, for our purposes, at some point arbitrarily close to merger. It may be interesting to note that the expression multiplying K_S takes into account effects of orbital precession (because it is non-vanishing if and only if the orbit precesses) and therefore represents physical phenomena neglected by previous fits.

4. Fitting procedure and results

The fitting of the out-of-plane recoil is in principle complicated because in addition to the overall factors K_2, K_3, and K_S (which are the same for any mass ratio or spins), any particular set of runs with the same initial separation and mass ratio (which we will term a "block") has idiosyncratic values of Φ_1 and Φ_2 that, although not fundamentally interesting, need to be fit to the data. Therefore, in the 17 blocks of data we fit, there are formally $3 + 2 \times 17 = 37$ fit parameters. In the eight data blocks for which $\alpha_1^{\perp} = 0$, Φ_1 never enters, and in the two for which $\alpha_1^{\perp} = \alpha_2^{\perp}$, $\Phi_1 = \Phi_2$. Therefore the actual number of fitting parameters is 27, but this is still large enough that a multiparameter fit would be challenging. Fortunately, each (Φ_1, Φ_2) pair only affects a single data block. We can therefore speed up the fitting, and incidentally concentrate on only the interesting parameters, if we (1) pick some values of K_2, K_3, and K_S that apply to all data blocks, then (2) for each data block, find the values of Φ_2 and possibly Φ_1 that optimize the fit, repeating this using new values of Φ_1 and Φ_2 for each block. This gives an overall fit for the assumed values of K_2, K_3, and K_S, having optimized over the uninteresting Φ parameters.

The fit itself needs to be performed assuming uncertainties on each of the numerical measurements of the kick. Each such calculation is computationally expensive and systematic errors are usually difficult to quantify, hence we do not have enough information to do a true fit. As a substitute, we assume that for each block of data, the uncertainty σ in each kick is either equal to a fraction (fixed for all blocks) of the maximum magnitude kick in the block, or to a fixed fraction of the individual kick itself. The former may be justified because some sources of error will be independent of the phase of the angles when the holes merge, but we note that the fit performed with the latter assumption (that the uncertainty equals a fractional error of each kick) yields very similar values for the fitting factors. As we do not

know what the actual fractional error is, in either case we adjust it so that for our best fit we get a reduced χ^2 of roughly unity (given our 95 data points and 27 fitting variables, this means we need a total χ^2 of about 68). We then evaluate every (K_2, K_3, K_S) triplet using $\chi^2 = \sum(\text{pred} - \text{kick})^2/\sigma^2$.

Minimizing χ^2 as calculated with respect to the maximum kick per block, we find that $\sigma^2 = 0.0005V_{\|,\text{max}}^2(\text{block})$ gives $\chi^2/\text{dof} = 1.0$. We note that this value for σ is comparable to the numerical error as measured by the difference in kicks computed at different resolutions, when available (e.g. for the new simulations presented here, or the $q = 0.25$ case presented in Table IV of Lousto & Zlochower (2009)). It is also worth noting that for the data from the new simulations, the uncertainty due to orbital precession, discussed in § 2, is less than $0.02V_{\|,\text{max}} < \sigma$, and therefore cannot significantly affect the fit. Our best fit is $K_2 = 30,540$ km s^{-1}, $K_3 = 115,800$ km s^{-1}, and $K_S = 17,560$ km s^{-1}, with 1σ ranges $28,900 - 32,550$ km s^{-1}, $107,300 - 121,900$ km s^{-1}, and $15,900 - 19,000$ km s^{-1}, respectively. We emphasize that all three coefficients are indispensable in obtaining a good fit; e.g. using the same definition of χ^2 as above, if $K_3 = 0$ then the best fit gives $\chi^2/\text{dof} = 2.0$, or if $K_S = 0$ then the best fit gives $\chi^2/\text{dof} = 1.6$.

Minimizing χ^2 as calculated with respect to each individual kick, we obtain similar results. In this case, $\sigma^2 = 0.0016V_{\|}^2$ gives $\chi^2/\text{dof} = 1.0$. Our best fit becomes $K_2 = 32,092$ km s^{-1}, $K_3 = 108,897$ km s^{-1}, and $K_S = 15,375$ km s^{-1}, in agreement with the above fit to within $\sim 5\%$ for K_2 and K_3 and $\lesssim 12\%$ for K_S.

In Table 4 we compare the predicted out-of-plane kicks with the measured ones for the entire data set. For fit #1 minimizing the error with respect to the maximum kick per block, of the 95 points, only 4 agree to worse than 10%, and all of those occur for kicks with magnitudes much less than the maximum in their data block. Therefore these could represent small phase errors rather than relatively large fractional velocity errors. Only 15 of the 95 points agree to worse than 5%. Fit #2 minimizing the error with respect to individual kicks performs even better in this regard, with only 10 points differing by more than 5% and only 2 points differing by more than 10%. In either case, the fits with the new ansatz are significantly better than either the simpler η^2 fit proposed by Lousto & Zlochower (2009) or the η^3 fit proposed by Baker et al. (2008).

Table 1. Recoil data from Lousto & Zlochower (2009) (set A), Baker et al. (2008) (set B), this work (set C), Dain et al. (2008) (set D), and Campanelli et al. (2007b) (set E). The spin angles ϕ_1 and ϕ_2 are in radians, and the recoil velocities $V_{||}$ are in km s^{-1}. The 8th column shows the result of a fit intended to minimize the error relative to the maximum recoil velocity per $(q, \alpha_1^{\perp}, \alpha_2^{\perp})$ triplet and the 9th column shows the relative error from that fit. The 10th column shows the result of a fit intended to minimize the conventional relative error and the 11th column shows the relative error from that fit. In both cases, the vast majority of points agree to well within 10%.

set	q	$\alpha_1^{\perp}$	$\alpha_2^{\perp}$	ϕ_1	ϕ_2	num. $V_{\|\|}$	fit $V_{\|\|}$ #1	$\|\Delta V/V\|$	fit $V_{\|\|}$ #2	$\|\Delta V/V\|$
A	0.125	0.000	0.751	0.000	1.571	-113.3	-118.5	0.046	-118.8	0.049
A	0.125	0.000	0.756	0.000	1.640	-101.9	-95.9	0.058	-96.5	0.053
A	0.125	0.000	0.761	0.000	0.223	-349.1	-354.5	0.016	-350.1	0.003
A	0.125	0.000	0.747	0.000	4.665	131.6	133.4	0.014	133.4	0.014
A	0.125	0.000	0.772	0.000	3.128	338.7	339.6	0.003	335.0	0.011
A	0.167	0.000	0.777	0.000	1.571	530.6	524.7	0.011	517.0	0.026
A	0.167	0.000	0.739	0.000	2.688	348.8	369.1	0.058	368.6	0.057
A	0.167	0.000	0.786	0.000	0.947	320.3	327.1	0.021	318.9	0.004
A	0.167	0.000	0.773	0.000	-1.509	-529.0	-532.0	0.006	-524.6	0.008
A	0.167	0.000	0.778	0.000	3.721	-130.8	-140.4	0.074	-133.5	0.021
A	0.250	0.000	0.779	0.000	1.571	-909.9	-908.1	0.002	-898.0	0.013
A	0.250	0.000	0.788	0.000	1.203	-815.9	-811.8	0.005	-801.1	0.018
A	0.250	0.000	0.760	0.000	0.080	56.3	52.2	0.074	56.1	0.004
A	0.250	0.000	0.787	0.000	4.463	866.0	858.4	0.009	847.7	0.021
A	0.250	0.000	0.751	0.000	3.041	-209.9	-209.2	0.003	-211.3	0.007
A	0.333	0.000	0.794	0.000	1.571	-1145.2	-1123.8	0.019	-1109.6	0.031
A	0.333	0.000	0.794	0.000	1.164	-832.5	-833.6	0.001	-819.3	0.016
A	0.333	0.000	0.754	0.000	0.082	397.7	387.9	0.025	391.8	0.015
A	0.333	0.000	0.795	0.000	4.466	981.7	968.3	0.014	953.8	0.028
A	0.333	0.000	0.751	0.000	3.017	-611.4	-603.8	0.012	-604.9	0.011
A	0.400	0.000	0.793	0.000	1.571	-1414.7	-1399.6	0.011	-1386.6	0.020
A	0.400	0.000	0.798	0.000	1.054	-1180.6	-1170.4	0.009	-1155.3	0.021
A	0.400	0.000	0.767	0.000	0.017	91.7	83.9	0.084	91.3	0.004
A	0.400	0.000	0.798	0.000	4.426	1338.6	1319.5	0.014	1304.8	0.025
A	0.400	0.000	0.760	0.000	2.980	-328.3	-321.0	0.022	-326.0	0.007
A	0.500	0.000	0.771	0.000	1.571	34.0	26.6	0.217	34.0	0.000
A	0.500	0.000	0.777	0.000	0.642	1261.7	1249.7	0.010	1244.4	0.014
A	0.500	0.000	0.800	0.000	-0.323	1528.2	1493.9	0.022	1479.7	0.032
A	0.500	0.000	0.764	0.000	4.320	-622.6	-603.4	0.031	-605.6	0.027
A	0.500	0.000	0.800	0.000	2.674	-1439.4	-1401.8	0.026	-1387.2	0.036
A	0.666	0.000	0.772	0.000	1.571	895.2	872.5	0.025	865.9	0.033

Table 1—Continued

set	q	$\alpha_1^{\perp}$	$\alpha_2^{\perp}$	ϕ_1	ϕ_2	num. $V_{\|\|}$	fit $V_{\|\|}$ #1	$\|\Delta V/V\|$	fit $V_{\|\|}$ #2	$\|\Delta V/V\|$
A	0.666	0.000	0.802	0.000	0.506	1699.3	1672.6	0.016	1662.6	0.022
A	0.666	0.000	0.798	0.000	-0.356	1025.0	1000.5	0.024	995.7	0.029
A	0.666	0.000	0.784	0.000	4.325	-1362.7	-1339.9	0.017	-1330.8	0.023
A	0.666	0.000	0.795	0.000	2.654	-823.9	-813.3	0.013	-809.8	0.017
A	1.000	0.000	0.800	0.000	1.571	1422.3	1422.6	0.000	1421.0	0.001
A	1.000	0.000	0.803	0.000	0.428	1009.3	990.5	0.019	980.9	0.028
A	1.000	0.000	0.785	0.000	-0.461	-246.0	-237.4	0.035	-245.3	0.003
A	1.000	0.000	0.805	0.000	4.222	-1479.9	-1468.0	0.008	-1461.9	0.012
A	1.000	0.000	0.786	0.000	2.581	390.5	380.5	0.026	387.8	0.007
B	0.333	0.200	0.022	0.000	3.142	49.0	50.5	0.031	52.0	0.062
B	0.333	0.200	0.022	5.498	2.356	48.0	48.1	0.003	48.9	0.018
B	0.333	0.200	0.022	4.712	1.571	17.0	17.6	0.034	17.2	0.012
B	0.333	0.200	0.022	0.000	0.000	114.0	114.3	0.003	115.2	0.011
B	0.500	0.200	0.050	0.000	3.142	-37.0	-38.2	0.033	-36.7	0.008
B	0.500	0.200	0.050	5.498	2.356	111.0	111.2	0.002	114.1	0.028
B	0.500	0.200	0.050	4.712	1.571	193.0	195.5	0.013	198.0	0.026
B	0.500	0.200	0.050	5.498	1.571	75.0	73.2	0.023	76.4	0.019
B	0.500	0.200	0.050	0.000	1.571	-55.0	-56.7	0.031	-55.0	0.000
B	0.666	0.200	0.089	1.047	4.189	-381.0	-382.9	0.005	-384.5	0.009
B	0.666	0.200	0.089	0.000	3.142	-135.0	-132.1	0.022	-133.4	0.012
B	0.666	0.200	0.089	5.498	2.356	168.0	165.3	0.016	165.2	0.017
B	0.666	0.200	0.089	4.712	1.571	364.0	365.9	0.005	367.0	0.008
B	0.769	0.200	0.118	0.000	3.142	-386.0	-387.2	0.003	-388.1	0.005
B	0.769	0.200	0.118	5.498	2.356	-525.0	-521.7	0.006	-521.2	0.007
B	0.769	0.200	0.118	4.712	1.571	-348.0	-350.6	0.008	-349.0	0.003
B	0.909	0.200	0.165	0.000	3.142	-542.0	-542.8	0.001	-542.4	0.001
B	0.909	0.200	0.165	5.498	2.356	-657.0	-656.3	0.001	-655.2	0.003
B	0.909	0.200	0.165	4.712	1.571	-384.0	-385.3	0.003	-384.1	0.000
C	0.674	0.367	0.236	2.513	5.655	-973.7	-944.4	0.030	-941.0	0.034
C	0.674	0.367	0.236	2.094	4.189	-442.9	-435.8	0.016	-431.8	0.025
C	0.674	0.367	0.236	2.094	0.000	-988.5	-934.6	0.055	-934.4	0.055

Table 1—Continued

set	q	$\alpha_1^\perp$	$\alpha_2^\perp$	ϕ_1	ϕ_2	num. $V_{\|\|}$	fit $V_{\|\|}$ #1	$\|\Delta V/V\|$	fit $V_{\|\|}$ #2	$\|\Delta V/V\|$
C	0.674	0.367	0.236	1.257	4.398	-361.7	-344.3	0.048	-339.4	0.062
C	0.674	0.367	0.236	0.000	4.189	374.2	331.6	0.114	336.5	0.101
C	0.674	0.367	0.236	0.000	3.142	749.0	731.6	0.023	731.2	0.024
C	0.674	0.367	0.236	0.000	2.094	672.4	682.2	0.015	677.3	0.007
C	0.674	0.367	0.236	5.027	1.885	826.1	796.5	0.036	791.4	0.042
C	0.674	0.367	0.236	4.189	2.094	619.7	603.0	0.027	597.9	0.035
C	0.674	0.367	0.236	4.189	0.000	-245.6	-246.4	0.003	-245.5	0.000
C	0.674	0.367	0.236	3.770	0.628	-237.7	-239.4	0.007	-242.1	0.019
C	0.515	0.367	0.177	2.513	5.655	-584.3	-578.0	0.011	-587.0	0.005
C	0.515	0.367	0.177	2.094	4.189	-123.0	-103.3	0.160	-118.1	0.040
C	0.515	0.367	0.177	2.094	0.000	-684.8	-683.1	0.002	-685.0	0.000
C	0.515	0.367	0.177	1.257	4.398	-16.9	1.0	1.059	-16.5	0.024
C	0.515	0.367	0.177	0.000	4.189	483.9	462.6	0.044	448.5	0.073
C	0.515	0.367	0.177	0.000	3.142	579.8	578.6	0.002	576.8	0.005
C	0.515	0.367	0.177	0.000	2.094	345.2	346.7	0.004	357.5	0.036
C	0.515	0.367	0.177	5.027	1.885	362.4	356.6	0.016	373.0	0.030
C	0.515	0.367	0.177	4.189	2.094	202.1	220.5	0.091	236.6	0.171
C	0.515	0.367	0.177	4.189	0.000	-231.1	-243.4	0.053	-239.4	0.036
C	0.515	0.367	0.177	3.770	0.628	-354.8	-358.2	0.010	-346.3	0.024
D	1.000	0.930	0.930	1.571	4.712	2372.0	2413.6	0.018	2423.7	0.022
D	1.000	0.930	0.930	1.833	4.974	2887.0	2972.2	0.030	2975.8	0.031
D	1.000	0.930	0.930	2.269	5.411	3254.0	3440.6	0.057	3432.8	0.055
D	1.000	0.930	0.930	3.176	0.035	2226.0	2390.4	0.074	2366.0	0.063
D	1.000	0.930	0.930	4.817	1.676	-2563.0	-2659.2	0.038	-2666.8	0.041
D	1.000	0.930	0.930	4.974	1.833	-2873.0	-2972.2	0.035	-2975.8	0.036
D	1.000	0.930	0.930	5.149	2.007	-3193.0	-3233.9	0.013	-3232.9	0.013
D	1.000	0.930	0.930	5.934	2.793	-2910.0	-3152.3	0.083	-3133.1	0.077
E	1.000	0.515	0.515	1.571	4.712	1833.0	1881.1	0.026	1875.6	0.023
E	1.000	0.515	0.515	0.785	3.927	1093.0	1079.6	0.012	1076.4	0.015
E	1.000	0.515	0.515	3.142	0.000	352.0	354.4	0.007	353.3	0.004
E	1.000	0.515	0.515	4.712	1.571	-1834.0	-1881.1	0.026	-1875.6	0.023

5. Ejection probabilities and discussion

One of the most important outputs of kick calculations and fits is the probability distribution of kicks given assumptions about the mass ratio, spin magnitudes, and spin directions. This distribution is critical to studies of hierarchical merging in the early universe (e.g., Volonteri (2007)) as well as to the gas within galaxies (Devecchi et al. 2009) and an evaluation of the prospects for growth of intermediate-mass black holes in globular clusters (Holley-Bockelmann et al. 2008). In Table 2 we show the results of our work (from fit #1), compared with the proposed fit formula of Campanelli et al. (2007a). It is clear that our work gives distributions very close to those of Campanelli et al. (2007a), with perhaps slightly smaller kicks because of the η^3 term we include.

In summary, we have demonstrated that a modified formula fits all available out-of-plane kicks extremely well. The wide range of mass ratios, spin magnitudes, and angles explores all the major aspects of parameter space for the out-of-plane kicks, and thus we do not expect new results to deviate significantly from our formula. The excellence of these fits suggests that the kick distribution is known to an accuracy that is sufficient for any astrophysical purpose.

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Table 1—Continued

set	q	$\alpha_1^\perp$	$\alpha_2^\perp$	ϕ_1	ϕ_2	num. $V_{\|\|}$	fit $V_{\|\|}$ #1	$\|\Delta V/V\|$	fit $V_{\|\|}$ #2	$\|\Delta V/V\|$
E	1.000	0.515	0.515	3.304	0.162	47.0	46.3	0.014	46.2	0.017
E	1.000	0.515	0.515	0.000	3.142	-351.0	-354.4	0.010	-353.3	0.007

Table 2. Fraction of kick speeds above a given threshold, compared with the results of Campanelli et al. (2007a) (CLZM). In all cases we assume an isotropic distribution of spin orientations.

Mass ratio and spin	Speed threshold	CLZM	This work
$1/10 \leq q \leq 1$, $\alpha_1 = \alpha_2 = 0.9$	$v > 500$km s^{-1}	0.364±0.0048	0.342526±0.00019
	$v > 1000$km s^{-1}	0.127±0.0034	0.120974±0.00011
$1/4 \leq q \leq 1$, $\alpha_1 = \alpha_2 = 0.9$	$v > 500$km s^{-1}	0.699 ± 0.0045	0.697818±0.00026
	$v > 1000$km s^{-1}	0.364±0.0046	0.353393±0.00019
$1/4 \leq q \leq 1$, $0 \leq \alpha_1, \alpha_2 \leq 1$	$v > 500$km s^{-1}	0.428±0.0045	0.415915±0.00020
	$v > 1000$km s^{-1}	0.142±0.0034	0.134615±0.00012

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